Dear Consortium Math Teachers,

We are all privileged to be part of the Consortium where we have the great opportunity to teach mathematics with great imagination and share its beauty and power with our students. We have the opportunity to present mathematics so that students can come to have a new emotion and appreciation for this wonderful discipline. We have the chance to challenge our students in new and exciting ways and have them begin to see they are capable of thinking deeply about mathematics.

This is why our PBAT work is so important. It is the culmination of each school's creative math program where students get to share their understanding of mathematics. The document you are about to read represents the ideas of teachers from almost every Consortium school. In five sessions we looked critically at our PBAT work and talked about how we can ensure that the work students do on a math PBAT is authentic and transforming. The PBAT experience in mathematics should be an experience a student never forgets because he or she had the opportunity to use his or her mind in deep and new ways that can make for true pride.

Finally, our goal is to have this document become central in all schools as they plan their PBAT work. That is why we want to hear from you about what is within the document. It should be stated that this document is fluid, always ready to be altered to meet the needs of our schools. It represents our thinking in February of 2016.

I would like to express my appreciation to Athena Leonardo (James Baldwin), Patty Fox (ICE), Wan Park (CSSJ), Gabriella Weisberg (Humanities Prep), Terry Weber (Urban Academy-retired) and Joe Walter (CPESS-retired) for their incredible work in bringing this document together.

Sincerely,

Jonathan Katz
Math Specialist for the Consortium

Quality and Criteria for a Mathematics PBAT

Introduction

A performance-based assessment task (PBAT) should be a summative, non-routine, complex problematic scenario that allows for multiple avenues of problem solving. A PBAT is a final task in which a student demonstrates his or her ability to think and reason mathematically. Prior to this PBAT all students should have had many experiences throughout his or her high school career in which they grappled with non-routine, complex, problematic scenarios (formative tasks), so that students are not surprised by the nature of its non-routineness. The PBAT may or may not be an open-ended task with multiple "correct" answers such as *Comparing and Contrasting* $\boldsymbol{\pi}$ *with* $\boldsymbol{\phi}$, or *Who is the best hitter is in Major League Baseball?,* or *Proving Gaussian's Idea on Summation*, or *Are Lotteries Fair?* but it should have the possibility of being thought about and approached in multiple ways. The task must be appropriately challenging and mathematically rich. The topic of a PBAT may be given to students by the teacher, but can also be co-generated with students.

A PBAT should reflect the highest level of mathematical understanding and thinking the student has displayed over his or her years at the school. Thus the content that is embedded in the PBAT should vary according to the student. Some students have shown a strong ability to think and reason mathematically and so the PBAT he or she is asked to work on should reflect that level of understanding. Other students come to this work with great trepidation and years of difficulty making sense of mathematics so the PBAT they work on should be challenging in terms of content yet appropriate for their level of understanding. All tasks should be geared towards promoting student autonomy and independent mathematical thinking.

Non-routine scenarios should considerably expand and extend problems that are routinely exhibited in textbooks. The scenarios should also situate students in an unfamiliar setting that forces students to adapt to a complex problem using mathematics. A complex problem requires multiple steps of connected mathematical concepts and procedures for it to be solved. A student must defend his/her process and solution using rigorous mathematical analysis.

The PBAT must be aligned to the five dimensions in the Math Consortium rubric. These five dimensions reflect aspects of the thinking process a student goes through when he or she solves a problem. These five dimensions have a fairly long history dating back to their presentation by NCTM in 1989. When creating a PBAT these dimensions need to be front and center in one's mind to insure that we are demanding deep mathematical thinking and reasoning from our students.

PBATs should require students to communicate their findings in a sophisticated mathematical paper and a presentation. To authentically assess students' thinking,

teachers may additionally require students to perform related mathematical task *on-demand* at the time of their presentations. Each task should provide an opportunity for students to earn an *Outstanding* mark in each category of the Consortium's rubric

In the following sections we will focus on areas that teachers felt were most important when thinking about the math PBAT work. The sections are: independent mathematical thinking, student metacognition, scripting and scaffolding and student choice and differentiation.

Independent Mathematical Thinking

An independent thinker is aware that there are multiple strategies to solving a problem. He or she should be able to make unprompted connections between different problems from his/her own mathematical experience and between conceptual ideas and procedural methods presented in their PBAT. An independent thinker shows a strong willingness to persevere with problems and understands the importance in defending and explaining one's reasoning. On the whole an independent thinker will take ownership of both the process and product in working on a PBAT.

A teacher's role is to provide guidance and encouragement, ask appropriate questions, celebrate small successes, build trust amongst students and assess readiness. The teacher's role is not to give answers or offer "too much" help. The thinking needs to be the student's thinking, not the teacher's thinking.

Revision is part of the writing process yet it can raise some issues. The main goal in revision is to insure that through the revision process the product remains truly the student's work. When a teacher asks a student to make revisions the student needs to be able to articulate what needs to be revised, why it needs to be revised, and how they intend to make the revisions. We need to insure that the revision process is based on the student's thinking and it contains their original ideas.

If we create a math program that has as its intention the development of independent mathematical thinking we have a greater chance of impacting the way our students grapple with the PBAT. As part of this math program students should expect on an everyday basis that they would be challenged to think mathematically. Students will develop independence if we give them many low stakes problem-solving experiences throughout their high school math classes. These experiences should be diverse including relevant and contextual problem solving. Students need to be challenged appropriately on an on-going basis with opportunities to reflect on their own development as problem solvers and thinkers.

Student Metacognition

Alan Schoenfeld, one of the leading thinkers on mathematical problem solving for over thirty years has talked about the importance of being able to look critically at one's own work as one solves a problem. Since knowledge about one's thought processes and developing the ability to self-regulate during problem solving are crucial for any students' development as a problem solver, it is important that students be able to discuss their thinking as they grapple with the PBAT. What might that look like in the student work?

Student metacognition can be articulated throughout the paper, when appropriate. Students might discuss:

> What approach to the problematic situation did you take?
> Why did you take that approach?
> How did you assess the validity of your approach?
> Did you need to rethink your approach? Why? What did you do?
> How did you become convinced that your thinking about the problem was correct and that your solution made sense?

Scaffolding and Scripting

It is appropriate to give different levels of scaffolding to different students as needed. However, the goal of the teacher is to minimize scaffolding, thus maximizing the students' independent thinking. Scaffolding can be useful if its purpose clearly is to enhance student thinking. Unfortunately scaffolding can have the opposite effect, forcing students to think in the proscribed way and thus interfering with the student's own original thinking and reasoning. We need as much as possible to see ourselves as facilitators only entering into a student's thinking process when necessary. If it appears that a student does not understand the mathematics needed to complete a PBAT task, this signals that this student must pause on the PBAT and review the content in a different context so that the PBAT remains valid.

Scripting is directing students through one or each step of a problem by either statements or guiding questions. This narrows the focus and prevents the student from thinking independently—a goal of Consortium mathematics. Students should have time to think for themselves and choose their own strategy or approach to solving a problem, even if the thinking is flawed or inefficient.

Scripting should not be used during PBAT work. When a teacher thinks scripting is

necessary for some students to do a PBAT does this signal that the PBAT is inappropriate for the student at this time?

Scripting might be useful in pre-PBAT math classes where students are learning how to develop as mathematical thinkers. This approach might model for students the types of questions they need to ask themselves as they develop as problem solvers and mathematical thinkers. But as a student moves along in his or her high school career the scripting should be lessened and finally eliminated from the work in mathematics.

Student Choice and Differentiation

Student choice is important for increasing engagement, personal connection, and ownership/accountability for students. Allowing for student choice also provides additional opportunities for critical-thinking, application, and original work.

We recognize that opportunities for student choice must be structured and differentiated. How do we insure that we are asking students to engage in a meaningful mathematics experience? How do we insure that the task the student is doing is appropriate for that student? We should keep this statement from George Polya in mind as we work with students.

> *If he challenges the curiosity of his students by setting their problems proportionate to their knowledge, and helps them to solve their problems with stimulating questions, he may give them a taste for, and some means of, independent thinking.*

To ensure rigor in PBAT assignments that are also unique for each type of student and learner, teachers must be explicit about what our math expectations are for each student. Those expectations should be aligned to the Consortium math rubric. In the Consortium we have the ability to know our students well. Teachers can then push each student beyond the basic set of expected understandings and skills to his or her highest level of mathematical thinking.

This expectation of choice and differentiation can seem challenging and overwhelming for a teacher. The purpose here is not to have each student work on a totally unique problem. But it is asking teachers to make sure that student choice and differentiation are central to the planning of this work so we are helping students to have an experience with their math PBAT that will be truly memorable and help them to become deeper mathematical thinkers.